

Advanced Algorithms

October 2, 2025

- You are running a Hot Tub production company.
- You can produce two types of hot tubs: Aqua-Spas and Hydro-Luxes.
- They require resources (pumps, labor, and tubing), and yield a certain profit

	<i>Aqua-Spa</i>	<i>Hydro-Lux</i>
<i>Pumps</i>	1	1
<i>Labor</i>	9 hours	6 hours
<i>Tubing</i>	12 feet	16 feet
<i>Price</i>	\$350	\$300

- You have 200 pumps, 1566 hours of labor, and 2880 feet of tubing.
- How many of each hot tub to produce if we want to maximize sales?

Logistics

- Exercise set 2
 - Due next Tuesday **in class**. On webpage
 - Submit your feedback about the course
- Assignment 2 will be released next Tuesday
 - Will be shorter
- No class next Thursday
 - I'm going to a conference :)
- Room change after Fall break

Linear Programming is in P

$$\text{max/min} \quad r_1x_1 + r_2x_2 + r_3x_3 + \cdots + r_nx_n$$

$$\text{subject to} \quad a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n \leq u$$

$$\vdots$$

$$b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_nx_n \geq v$$

$$\vdots$$

$$c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n = w$$

$$\vdots$$

Integer Linear Programming . . .

$$\text{max/min } r_1x_1 + r_2x_2 + r_3x_3 + \cdots + r_nx_n$$

$$\text{subject to } a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n \leq u$$

$$\vdots$$

$$b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_nx_n \geq v$$

$$\vdots$$

$$c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n = w$$

$$\vdots$$

x_i is an integer value for all $i \in [n]$

Is NP-hard

Why?

Integer Programming formulation:

x = number of Aqua-Spas to produce

y = number of Hydro-Luxes to produce

Maximize: $350x + 300y$

Subject to:

$$x + y \leq 200 \quad (\text{pumps})$$

$$9x + 6y \leq 1566 \quad (\text{labor})$$

$$12x + 16y \leq 2880 \quad (\text{tubing})$$

$$x, y \geq 0 \quad (\text{non-negativity})$$

x, y integer

Linear Programming relaxation:

x = number of Aqua-Spas to produce

y = number of Hydro-Luxes to produce

Maximize: $350x + 300y$

Subject to:

$$x + y \leq 200 \quad (\text{pumps})$$

$$9x + 6y \leq 1566 \quad (\text{labor})$$

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$$x, y \geq 0 \quad (\text{non-negativity})$$

~~x, y~~ integer

How can this be
used in an
algorithm?

New Scenario

	<i>Aqua-Spa</i>	<i>Hydro-Lux</i>
<i>Pumps</i>	1	1
<i>Labor</i>	9 hours	6 hours
<i>Tubing</i>	12 feet	16 feet
<i>Price</i>	\$350	\$300

- You have 200 pumps, 1566 hours of labor, and 2880 feet of tubing.
- You have hired exactly 1566 hours, have to pay \$8 for over time or idle time.

How to model this?